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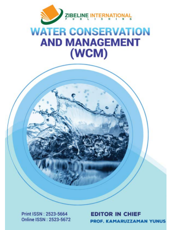

RESEARCH ARTICLE

AN ANALYSIS OF WATER FLOW IN SUBSURFACE ENVIRONMENT BY USING ADOMIAN DECOMPOSITION METHOD

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ABSTRACT

Fluid flow is major part of subsurface water environment. This study considered the steady, flow of an incompressible, isothermal and homogeneous third grade fluid between two parallel plates. The resulting equation is solved with the help of Adomian Decomposition Method (ADM) and critical analysis of results indicates that velocity of fluid depends on several parameters.

KEYWORDS

Plane Poiseuille flow, Adomian Decomposition Method, Third grad flow.

1. INTRODUCTION

This paper will consider some issues closely related between mathematics and subsurface water environment [1]. They come out as an outcome of treating with random mathematical formulations in representing this phenomenon. It resulted that the mathematical and that physical formulations joint together to a single point, and hence showing a lot of problems in using mathematical and material formulations in modeling of environmental issues. In both cases, problem have come out of an epistemological level and because of careless treatment of some non-linear and complex real-world problems. Although in some cases when the problem was physically quite simple, the mathematical picture presenting that physical problem was not always on that right way. In short account, one can observe that issues always occur directly or indirectly (a) physical problems and nonlinear (b) relation between environment and mathematics. Looking upon the first question under discussion, our observation shows that "the nonlinearity can uncover a lot, but it does not allow much to be discovered" (D.M). The later question under discussion was discussed by Winger (1960) and elaborated the effectiveness of mathematics in different fields [2].

Water is main part of our environment and is cause of life on earth. Environmental flow is term of environmental sciences that describes the quality [3], timing of water flow and quantity, that is necessary to maintain freshwater on all ecosystems.

There are different types of water flow mathematically. Fluid flow was used instead of water to make it more general. Remember that fluid is both, liquid and gases. In depth, fluid may be Newtonian or Non-Newtonian.

Much importance and attention were received by non-Newtonian fluids [4] than that of Newtonian fluids. It was because of various technological and industrial uses of Non-Newtonian fluids. When the non-Newtonian fluids are modeled, the resulting differential equations are mostly of higher degree nonlinear and highly complicated. These nonlinear equations were solved by different methods, one of these methods is Adomian Decomposition Method [4]. A non-Newtonian can be distinguished from a Newtonian fluid by some of its physical qualities, like it's viscosity. The viscosity of non-Newtonian fluid is not constant on a particular pressure and temperature. The viscosity may change on velocity, flow geometry, flow conditions and some other factors.

Types of flow:

Flow of a fluid can be steady or unsteady, depending on the it's velocity [5]:

- **Steady**. In steady fluid flow, the velocity of fluid is does not change at any point.

- **Unsteady**. In unsteady fluid flow, the fluid's velocity may change between any two points.

- **Compressible**. You can compress the fluid if it is a compressible fluid. Gases (also considered a fluid) are very compressible.

- **Incompressible**. Liquids are usually nearly incompressible fluid.

- **Rotational**. If a fluid is spinning about its axis, it is called a rotational flow
- **Irrotational**. It is a type of flow in which every element of the moving fluid undergoes no net rotation.

- **Laminar flow**. If every particle of a fluid follows exactly smooth path and never cross other particle's path, this situation is called laminar flow.

- **Turbulent Flow**. If there is no particularly defined path for particles and they are flowing randomly, this is called turbulent

2. PLANE POISEUILLE FLOW OF A THIRD-GRADE FLUID BETWEEN COLLATERAL PLATES

Steady plane Poiseuille flow of third grade [6], incompressible, homogeneous and isothermal fluid between two collateral plates which are separated by some finite space. The developed second order nonlinear equation is solved by Adomian Decomposition Method and solution indicated that the velocity profile in x-direction.

The basic laws which an incompressible fluid follows are the law of conservation of mass, law of conservation of momentum and law of conservation of energy are given by [7]

$$(\nabla \cdot \mathbf{u}) = 0 \tag{1}$$

$$\rho\left(\frac{D\mathbf{u}}{Dt}\right) = \rho\mathbf{f} + \nabla \cdot \mathbf{T} \tag{2}$$

The constitutive equation used to be obeyed for a third-grade fluid is

$$\mathrm{T} = -\rho\mathrm{I} + \mu\mathrm{A}_1 + \alpha_1\mathrm{A}_2 + \alpha_2\mathrm{A}_1^2 + \beta_1\mathrm{A}_3 + \beta_2(\mathrm{A}_1\mathrm{A}_2 + \mathrm{A}_2\mathrm{A}_1) + \beta_3(tr\mathrm{A}_2)\mathrm{A}_1 \tag{2.1}$$

Where

$$A_1 = (\nabla V)^T + \nabla V \tag{2.2}$$

$$\mathrm{A}_n = \frac{D\mathrm{A}_{n-1}}{Dt} + \mathrm{A}_{n-1}\nabla V + (\nabla V)^T \mathrm{A}_{n-1}, \qquad n = 1,2,3,.. \tag{2.3}$$

Considered a third-grade fluid flowing between two plates placed horizontally at distance (h) apart. For simplicity it was preferred cartesian coordinates system with y-axis normal to flow and x-axis parallel to flow. For time dependent, uni-dimensional and nonlinear flow was defined velocity field as:

$$V = [u(y),0,0] \qquad S = S(y) \tag{3}$$

Or we can write the velocity profile as

$$\nabla V = \begin{pmatrix} 0 & \frac{du_x}{dy} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.1}$$

Taking transpose of equation (3.1) we get,

$$(\nabla V)^T = \begin{pmatrix} 0 & 0 & 0 \\ \frac{du_x}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.2}$$

Using equation (3.1) and equation (3.2) in equation (2.2) we get

$$A_1 = \begin{pmatrix} 0 & \frac{du_x}{dy} & 0 \\ \frac{du_x}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.3}$$

In the same way, all components of equation (2.1) can be calculated. So equation (2.1) becomes

$$\mathrm{T}_{xy} = T_{yx} = \left(\frac{du_x}{dy}\right) + \beta_2\left(\frac{du_x}{dy}\right)^2\left(\frac{du_x}{dy}\right)^3 + \beta_3^2\left(\frac{du_x}{dy}\right)\left(\frac{du_x}{dy}\right)^2 \tag{3.4}$$

$$\mathrm{T}_{yy} = 2\alpha_1\left(\frac{du_x}{dy}\right)^2 + \alpha_2\left(\frac{du_x}{dy}\right)^2 \tag{3.5}$$

Where

$$\mathrm{T} = \begin{pmatrix} \mathrm{T}_{xx} & \mathrm{T}_{xy} & \mathrm{T}_{xz} \\ \mathrm{T}_{yx} & \mathrm{T}_{yy} & \mathrm{T}_{yz} \\ \mathrm{T}_{zx} & \mathrm{T}_{zy} & \mathrm{T}_{zz} \end{pmatrix} \tag{3.6}$$

So, the non-zero component of momentum equation are

$$0 = \frac{\partial P}{\partial x} + \frac{d}{dx}\left[\mu\left(\frac{du_x}{dy}\right) + 2(\beta_2 + \beta_3)\left(\frac{du_x}{dy}\right)\left(\frac{du_x}{dy}\right)\right] \tag{3.7}$$

$$0 = -\frac{\partial P}{\partial y} + \frac{d}{dy}\left[2\alpha_1\left(\frac{du_x}{dy}\right)^2 + \alpha_2\left(\frac{du_x}{dy}\right)^2\right] \tag{3.8}$$

$$P^* = -P + 2\alpha_1\left(\frac{du_x}{dy}\right)^2 + \alpha_2\left(\frac{du_x}{dy}\right)^2 \tag{3.9}$$

Cancelling the zero components, and rearranging equation (3.9), we get

$$\frac{d^2u_x}{dy^2} = \frac{1}{\mu}\frac{\partial P^*}{\partial x} - \frac{6\beta}{\mu}\left(\frac{du_x}{dy}\right)^2\frac{d^2u_x}{dy^2} \tag{4}$$

Where $\beta = \beta_3 + \beta_2$. Introducing dimensionless parameters in equation (4)

$$u^* = \frac{u}{U},\ x^* = \frac{x}{h},\ y^* = \frac{y}{h},\ \lambda^* = \frac{U^2\beta}{\mu h^2},\ p^* = \frac{hp}{\mu U}$$

And dropping *, we get

$$\frac{d^2u}{dy^2} + 6\lambda\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2} = \frac{dP}{dx} \tag{5}$$

The associated boundary conditions of the problem are as follow

$u'(y) = 0$ at $y = 0$ and
$\mathrm{u(y)} = 0$ at $y = 1$

Solving this equation with the help of Adomian Decomposition method up to second order [8-10]:

$$u_y = \frac{1}{2}\frac{dP}{dx}\left(y^2-1\right)-\frac{\lambda}{2}\left(\frac{dP}{dx}\right)^3(y^4-1)+2\lambda^2\left(\frac{dP}{dx}\right)^{53}(y^6-1) \quad (5.1)$$

3.RESULTS AND DISCUSSION

The flow of a steady, isothermal, homogeneous and incompressible third grade fluid is investigated between two collateral plates placed apart from each other at a finite distance. The developed second order nonlinear ordinary differential equation is solved by using Adomian decomposition method. Mathematical results show that the velocity profile is dependent on non-Newtonian parameter λ . The increased value of λ reduces the velocity of fluid, which shows that fluid shear thickening occurs for the increasing values of λ . However, the velocity profile for Newtonian case can be retrieved for λ =0.

4. **CONCLUSION**

It was considered non-Newtonian fluid flow in subsurface water environment between two collateral plates. Plane Poiseuille flow of a third-grade fluid between collateral plates reflects that the velocity of fluid is dependent on the non-Newtonian parameter. In this study non-Newtonian parameter was not specify. Because it may be different for different cases. In our case, the subsurface fluid is suspension of water and other particles (sand, micro-organisms). These particles are the cause of non-Newtonian behavior of fluid and are the main non-Newtonian parameter in this study. Greater the non-Newtonian parameter, lower the velocity and vice versa.

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